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HOW LONG DOES IT TAKE FOR YOUR MONEY OR DEBT TO DOUBLE? EXPLAINING THE RULE OF 72 TO STUDENTS

¿CUÁNTO TIEMPO TARDA EN DUPLICARSE SU DINERO O DEUDA? EXPLICANDO LA REGLA DEL 72 A LOS ESTUDIANTES

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ABSTRACT

The 'rule of 72' provides a useful approximation of when an investment or debt will double. Students can apply it for an estimate, avoiding mistakes later when using technology for a precise answer. On standardized tests, moreover, such devices may be disallowed. In job interviews, too, quickly approximating the doubling answer demonstrates the impressive problem-solving ability. Illustrations abound online and in traditional media showing how to use it. This is not the same as explaining why it works or the limitations. Inquiring students want to know. This paper combines familiar territories in math and application to provide a relatively simple mathematical explanation.

KEYWORDS

management education, economics education, investments, money management, financial planning, economic growth

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RESUMEN

La "regla del 72" proporciona una aproximación útil de cuándo se duplicará una inversión o una deuda. Los estudiantes pueden aplicarlo para una estimación, evitando errores posteriores al usar la tecnología para una respuesta precisa. En las pruebas estandarizadas, además, dichos dispositivos pueden no estar permitidos. En las entrevistas de trabajo, también, aproximar rápidamente la respuesta duplicada demuestra la impresionante capacidad de resolución de problemas. Abundan las ilustraciones en línea y en los medios tradicionales que muestran cómo usarlo. Esto no es lo mismo que explicar por qué funciona o cuáles son sus limitaciones. Los estudiantes curiosos quieren saber. Este trabajo combina territorios familiares en matemáticas y aplicaciones para brindar una explicación matemática relativamente simple.

PALABRAS CLAVE

educación en administración, educación en economía, inversiones, administración del dinero, planificación financiera, crecimiento económico

INTRODUCTION

The ‘rule of 72’ determining the time required for an investment to double is a well-known approximation used in solving economics, business and finance problems. Using this rule if you can do simple arithmetic, no calculator or computer is required to quickly obtain an estimate of the answer to appealing questions as ‘if I make a promising investment, how long will it take my money to double?’ Or more optimistically, to triple? The also rule applies to debt growth concerns as ‘how fast will that outstanding credit card balance double? Really that soon?’ The objectives of this research are to: (1) provide a brief review of the history and popularity of the rule; (2) show how to apply it; (3) explain the mathematical rationale behind the rule; (4) determine how good an approximation it provides; (5) consider useful variants; (6) note when its use is appropriate; and (7) indicate some important new or reemphasized uses.

HISTORY AND APPLICATION

Lewin (2019) notes examples of the application of the rule of 72 date back to Pacioli (1494) i but more recently include publications by Slavin (1989), Morris and Lerro (1995), and popular textbooks in economics by McConnell, Brue and Flynn (2021) and personal finance by Garman and Forgue (2018). ii Online articles, too, in Investopedia (2020), Wikipedia (2021) and dozens of blogs posted by financial advisory organizations and firms (e.g. TIAA (2017), Saleta (2019) show how it can be easily used by individuals in planning and investing for large future expenses including retirement, home ownership, and spending on children’s education.

Here’s an example. Suppose Marla and Johan have need to make a quick financial decision. If they invest $25,000 in a friend’s business and earn an anticipated 9% annual return, how long (abstracting from risk) will it take for their money to double? To apply the rule of 72, divide that number by the rate of return expressed as a digit, i.e. the percentage return on investment multiplied by 100. The answer is approximately 72/9 = 8 years. Close to the exact answer. Another example uses the same formula but refers to the relationship between rate of inflation and the future price level. If that rate is nine percent, the price level can be expected to double in eight years.

'Explanations' of the rule like the last illustration are available online or in printed media. These show how to apply it (as above), not so much in answering 'where does it come from?' A reason for the omission, as Morris and Lerro (ibid.) note, is that the mathematics are challenging. But while the mathematics behind the rule can indeed become very complicated if a more accurate answer is sought (Majahan (2021), many university students have seen the basic math necessary before. iii My experience in business, finance, and economics classes is that several students ask me to explain the logic (mathematics) of the rule (or its origin). They emphasize they are more likely to remember and use it if they are comfortable with the arithmetic derivation. To this end, I simply adapt and attempt to clarify the explanation of simplifying without oversimplifying the math behind the rule.

DERIVING THE RULE

The amount of time it takes an investment to double depends upon the interest rate (known in this context as the 'rate of return (r)') using the 'future value formula.' In the version below, future value (FV) is equal to twice (doubling) the amount invested ('principal' of P):

[1] FV = P(1+r)n formula for future value

So 2P = P(1+r)n

Now dividing by P, we have

2 = (1+r)n

This can be solved for n using natural logarithmsiv as ln(2) = n*[ln(1+r)], giving us

[2] n = ln(2)/ln(1+r)

To simplify the math, the natural logarithm of 2 can be expressed to three decimal places as .693, a constant, so only the denominator needs to be determined for a given rate of return. That can be a challenge for those unfamiliar with natural logs. While a calculator or table of values of ln(1+r) can be used, that defeats the purpose of a quick answer without such tools. This is where a well-known approximation comes in, that the value of ln(1+r) is approximately equal to the rate of return if that rate is not too high,v or

[3] ln(1+r) $\approx r$

and the rule of 72 becomes

[4] n = ln(2) / r

As show later, this approximation is almost exact at just under eight percent, but seems reasonable for rates between five percentage points lower or higher than that, beyond which the error magnifies. vi

Simplifying further, the natural of 2 to three decimal places is .693, so we can express the doubling time of an investment or debt as:

[5] n = .693/r or more popularly as

n - 69.3/r*100.

When applied in this manner, the name 'rule of 69.3' has been used.vii Since the numerator is almost exact, it is the approximation in the denominator that is responsible for the rule of 72 or the rule of 69.3 to be considered an approximation. Of course, doing the arithmetic quickly for either rule without a computational device is not easy because of the value in the numerator. To expedite, that value can be simplified further, as .7. Multiplying the numerator and denominator by 100, we have the doubling time as 70 divided by r expressed as a digit. This is called the 'rule of 70.' viii The time required for an amount to triple in value can also be obtained by replacing the numerator of ln(2) with the numerator of ln(3). That value to three decimal places is 1.099, which when multiplied by 100 is very close to 110. The approximate time for tripling is 110/(r x 100). Trouble with doing arithmetic in your head? Simplify by first dividing 100 by r and then divide 10 by r and add the two quotients. So for an r of 5%, for example, 100/(5% x 100) equals 20 while ten divided by 5% times 100 equals 2. So the time required for tripling is approximately twenty-two years.

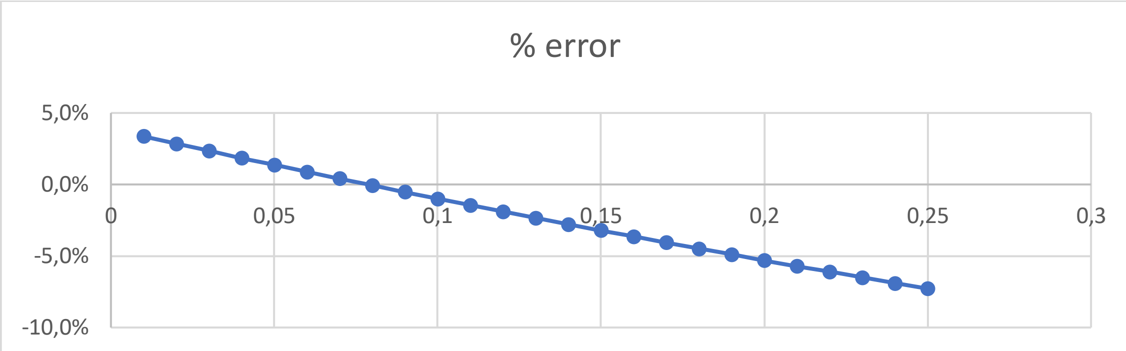
The more popular version, however, is the 'rule of 114', which is 114/(r x 100). For the problem just discussed, the approximate time for tripling is 22.8 years. The more precise answer using the original formula with ln(3) is 22.52 years, so the rule of 114 provides a little closer solution than using 110 in this example. Despite the imprecision in the numerator, the rule of 114 provides greater overall accuracy when used with the approximation of r rather than ln(1+r), similar to the rule of 72. The relative accuracy of all rules for doubling is discussed in the next section.

Rules can be easily derived for the time required for quadrupling, quintupling, and so on by simply replacing the ln(2) in the above by the ln of 4, 5, or higher values. in the equations above.

DETERMINING THE ACCURACY OF THE RULE OF 72

One way to show the accuracy of the Rule of 72 for different annual interest rates is to focus on the approximation error. That is defined here as the difference between the exact answer using equation (1) and the answer provided by the Rule of 72. The chart below then expresses the approximation error as a percentage of the exact answer. Thus, the percentage error due to approximation equals (exact answer – approximation) divided by the exact answer and appears on the vertical axis. The horizontal axis indicates the annual interest rate paid (the rate of return) or charged.

Chart 1. Percentage Approximation Error Using the Rule of 72 for Annual Interest Rates



As evident in the chart, beginning with an interest rate of 1% there is an overestimate using the rule of 72 which declines initially as the interest rate increases. A near-zero error is reached at an interest rate of eight percent. Beyond that interest rate, the approximation error is negative because the exact value is less than the estimated value. In terms of absolute value (ignoring the negative sign), the percentage error increases as the interest rate increases.

Depending upon how accurate an approximation is needed, you may decide to use the rule for interest rates over the range discussed above. If your error tolerance is high, you need not follow a customary guideline (up to twelve percent interest ix). For it might be in a demanding job interview in which the interviewer is more interested in your ability to quickly arrive at a reasonable answer, rather than expect either precision or vagueness. The forecast error rate is less than eight percent even at the top of the annual interest rate shown below (25%).

VARIANTS OF THE RULE OF 72 – THE RULE OF 70 AND THE RULE OF 69.3

Two variants of the rule of 72, the rules of 70 and 69.3, were noted earlier. How accurate are these variants, and which rule is most appropriate for use in particular circumstances? To investigate the relative accuracy of the three doubling rules, Table 1 shows a comparison of the exact value (equation [1] to two decimal places) and approximations using each rule for annual interest rates of 1% to 13%. The best estimate for each interest rate is shown in bold. For a rate of 1%, the rule of 69.3 is the most accurate. For rates in the 2-4% range, the Rule of 70 is most accurate. But for the remaining nine interest rates, the Rule of 72 is most accurate. Indeed, the relative inaccuracy (exact doubling value minus estimate) of the other two rules compared to the Rule of 72 grows as the interest rate further rises.

Table 1. Determining Which Rule Works Best for Different Annual Interest Rates

		Rule		
Interest rate	Exact value	72	70	69.3
1%	69.66	72.00	70.00	**69.30**
2%	35.00	36.00	**35.00**	34.65
3%	23.45	24.00	**23.33**	23.10
4%	17.67	18.00	**17.50**	17.33
5%	14.21	**14.40**	14.00	13.86
6%	11.90	**12.00**	11.67	11.55
7%	10.24	**10.29**	10.00	9.90
8%	9.01	**9.00**	8.75	8.66
9%	8.04	**8.00**	7.78	7.70
10%	7.27	**7.20**	7.00	6.93
11%	6.64	**6.55**	6.36	6.30
12%	6.12	**6.00**	5.83	5.78
13%	5.67	**5.54**	5.38	5.33

ACCURACY OF DIFFERENT RULES WHEN FREQUENCY OF COMPOUNDING IS INTRODUCED

When interest is compounded more frequently than once a year, as is often the case with credit cards, personal loans, corporate and government bonds and certificates of deposit the annual interest rate is not an appropriate measure of the effective interest rate. Notably, interest is often charged with different frequencies among several forms of debt, such as semiannually, quarterly, monthly, daily, and even continuously. This can also apply to investments. x Depending on the frequency of compounding, the time it takes for and investment outstanding balance to double can be determined using a modification of the exposition of the rules for doubling, as discussed above.

For greater accuracy, the annual equivalent interest rate that is paid is usually called in finance and economic texts the 'effective interest rate.' The formula defining that rate (ieff) is

[6] $ieff = (1+ i/f)f - 1$

where f is the frequency of compounding during the year, and i is the nominal (annual) interest rate charged.

As either the nominal rate rises or the frequency of compounding occurs more often, the difference between the effective rate and the nominal rate grows. For a nominal rate of 14% compounded daily, for example, the effective rate you are paying on your outstanding debt is 15.02%. For a higher nominal rate of 18%, charged on outstanding balances for some credit cards, the effective rate is 19.72%. These differences between nominal and effective interest rates indicate a substantial underestimation of interest if the nominal annual rate is used. But does this imply that the effective rate must be calculated, ditching the quick response to the doubling question, or is an approximation using the rule of 72 or another rule using the nominal rate sometimes acceptable?

To this end, Table 2 below compares the accuracy of predicted doubling time using the three different rules applied to daily interest rates. The exact value is the benchmark and is calculated by substituting the effective interest rate (far right column) for r in equation (2) above. The other three columns apply to doubling estimates derived from rules of 72, 70, and 69.3, in which the interest rate used is the nominal rate. Results shown apply to daily compounding, but very similar results (not shown) arise for quarterly compounding.

The main finding is that the rule of 69.3 approximates the exact value very well for the interest rates shown. In contrast to the earlier results using annual interest rates, the least accurate approximations follow from using the rule of 72. The accuracy of the rule of 70 falls in between, but some may prefer to use it because it is easy to calculate without tools and provides results not far off the rule of 69.3 mark.

Table 2. Accuracy of rules for daily interest compounding

		Rule			
Nominal rate	Exact value	72	70	69.3	Effective rate
2%	34.66	36.00	35.00	**34.65**	2.02%
4%	17.33	18.00	17.50	**17.33**	4.08%
6%	11.55	12.00	11.67	**11.55**	6.18%
8%	8.67	9.00	8.75	**8.66**	8.33%
10%	6.93	7.20	7.00	**6.93**	10.52%
12%	5.78	6.00	5.83	**5.78**	12.75%
14%	4.95	5.14	5.00	**4.95**	15.02%
18%	3.85	4.00	3.89	**3.85**	19.72%

CONCLUDING REMARKS – WHEN IS THE RULE MOST HELPFUL?

The analysis has considered the accuracy of the rule of 72 over an interest rate range (<14%) used in many economics and finance analyses. The rule has many applications in educational and practical finance applications. In problem-solving exercises students can apply it for an estimate, avoiding mistakes later from misusing technology. On standardized tests involving multiple choice or true-false questions, technology devices may be disallowed and the rule provides an efficient way of selecting the appropriate answer. In job interviews, too, quickly approximating the doubling answer demonstrates the impressive problem-solving ability many interviewers are looking for.

These skills can be quite important beyond formal education. This includes participating in business or government meetings involving investment decisions, advising clients with limited financial backgrounds or interesting others on planning for life cycle events of school expense, buying a home, retirement saving, and quantifying the burden of future outstanding debt. This is evident in the large number of informational blogs and videos available online and in printed media.

Following one of the many illustrations available for applying these rules, however, is not the same as promoting and retaining an understanding of where they come from, why they work, or their limitations. This knowledge can elevate those who can explain or show why the rule works and may be more apt to apply them in appropriate situations when it matters most in their careers.

It is important to recognize that the rule is not a substitute for exact determination when accuracy is of paramount concern, as would be the case for example in a legal (contractual) proceeding in which exact amounts of funds change hands. Students in advanced classes in finance or the sciences (the rule has been used in physics, biology) should be especially aware the rule has accuracy and applicability limitations. In finance, the rule is not appropriate for situations more complicated than a lump sum payment at the end, such as payments over multiple periods over time, as would be the case, say, for an annuity. The rule can be modified to deal with frequency of interest compounding during a year by using the effective interest rate (formula shown above), but that involves some complication. Interest rates outside that range have not been specifically considered, but it was noted that the accuracy of the rule diminishes as the interest rate rises.

Comparing the basic rule to its variants, the rules are somewhat different in terms of ease of usage and accuracy in different contexts. To avoid confusion from conflicting claims about which rule is overall best, students could be well advised to learn when to apply each. The analysis presented above suggests that the original (500+ years old) rule of 72 is easy to use and works best for the range of annual interest rates commonly used in economics and finance classes. The rule of 69.3, though, shines when considering the daily frequency of compound interest, again over the range of interest rates considered herein. However, not much accuracy is lost by switching to the rule of 70, which is easy to calculate and almost rivals in accuracy the more demanding to use rule of 69.3 for compound interest.

While the focus of the paper is on explaining the time to doubling, it is useful to mention there is a related application of the rule of 72 that has not been considered. Historically (see Lewin, ibid) the rule was sometimes used to determine the rate of return on an investment that is promised to double in value as of a particular date. Algebraically rearranging the order of terms in the rule (from $n = 72/r$) to $r = 72/n$ we can approximate the rate of return on a promised lump sum amount from an initial investment. This enables a quick approximation of the effect of a later maturity date on the rate of return. It more generally provides a ready (approximate) comparison of alternate investments of different amounts and date of anticipated lump sum returns in terms of the rate of return. It does not apply, however, to streams of returns over time. Future research can focus on the accuracy of the rule and its variants in this regard.

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[i] Lewin notes Pacioli's book 'famous for the first comprehensive treatise on book-keeping… has compound interest content… Like the ancient Babylonians before him, Pacioli is interested in the amount of time it takes for a loan to double in value, but unlike the Babylonians he proposes a general rule which he considers would be sufficiently accurate … This is the rule of 72.' Lewin adds 'Naturally, this rule is only an approximation, but it is quite a good one for interest rates between 3% and 12%.'

[ii] Morris and Lerro also applied the rule of 72 formula in reverse order to determine the rate of return when you know the date the investment will double. McConnell, et.al. used the rule in the context of promoting more rapid rate of economic growth. Garman and Forgue presented the rule early in their text as very useful in practical situations but an exact formula explained later in their text should be used when more precise answers are desired. Ovaska and Summell [2017] noted 'Though economic growth and personal finance are not commonly taught together, this paper shows that these topics can complement each other…'

[iii] Individuals who don't like math or have been away from it for quite a while may feel otherwise. Non-students may also not be so inclined. They can still apply the rule using simple arithmetic.

[iv] All logarithms are exponents to a stated base. In natural logarithms that base is approximately 2.718 which has some superior properties when discussing interest rates compared to common logarithms which use a base of 10.

[v] Some actual values of paired combinations of r in percentage terms and ln(1+r) in decimal terms are (4%, .039), (5%, .049), (6%,.058), (7%,.068), (8%,.077), (9%, .086), (10%, .095), and (11%, .104).

[vi] The rule is based on a linear approximation of a nonlinear function, and mathematical studies (for example, Morris and Lerro, ibid., who also cite Younger (1989) have indicated the extent to which the error of the linear approximation grows as the rate increases.

[vii] In a recent article Majahan (2020) used advanced mathematics to imply that the rule of 69.3 is mostly accurate when using continuous compounding (infinite frequency). The context was in the field of physics and did not focus upon the range of interest rate or time duration values commonly considered in finance or economics.

[viii] McConnell et. al. (ibid) and Slavin (ibid) use the rule of 70 rather than 72 in applications in their text.

[ix] Lewin, ibid.

[x] The rate a borrower will pay is not the same as the 'Annual Percentage Rate (APR).' That term was legally established in the United States (Truth in Lending laws) to make it easier for borrowers to compare interest rates which previously could be expressed in different time formats. If the interest rate is one percent per month, for example, the APR is 1% times twelve, or 12%. But this is an underestimate of the actual effective annual rate a borrower would pay, with underestimation increasing as the frequency of compounding rises.